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DETERMINATION OF THE EQUIVALENT DAMPING CONSTANT OF A PARALLEL ASSEMBLY OF ELASTIC SPRINGS

BY

EUGEN CORDUNEANU*

“Gheorghe Asachi” Technical University of Iaşi,
Department of Theoretical Mechanics

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Abstract. In this paper, a dynamical system composed by a solid mass attached at two elastic springs bounded in a parallel assembly is studied. An equivalently damping constant of the assembly in a free vibrations regime (mechanical vibrations) is obtained.

Key words: elastic spring, damping constant.

1. Introduction

One considers two helicoidally elastic springs with the elastic constants K_1, K_2 and damping constants C_1 and C_2, respectively. The springs are bounded in a parallel assembly in vertical position (Fig. 1) with the mass m attached at the extremities of springs, which have the same lengths under the action of this weight.

Let l be the common length of the springs and l_{10}, l_{20} the lengths of springs in undistorted state.

The characteristics of the mechanical system composed by these springs and mass satisfied the condition.

*Corresponding author; *e-mail*: ecorduneanu@yahoo.com

$$(C_1 + C_2)^2 < 4(K_1 + K_2)m\,. \tag{1}$$

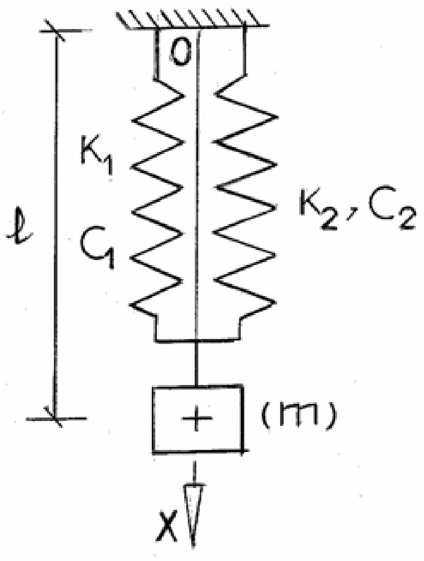

Fig. 1 − Parallel assembly of elastic springs.

2. Contents

If the system is taken out from the static equilibrium position, the dynamical equation for the attached mass is:

$$m\ddot{x} = mg + F_{e_1} + F_{a_1} + F_{e_2} + F_{a_2}\,, \tag{2}$$

where x is the coordinate, g − the gravitational acceleration, F_{e_1} − the elastic force of first spring, F_{a_1} − the damping force of first spring, F_{e_2} − the elastic force of second spring and F_{a_2} − the damping force of second spring.

The elastic forces are given by Hooke's relation (Buzdugan *et al.*, 1979; Teodorescu, 1984; Teodorescu, 1988; Teodorescu, 1997):

$$F_{e_1} = -K_1(x - l_{10})\,; \quad F_{e_2} = -K_2(x - l_{20}) \tag{3}$$

The damping forces are given by (Mangeron & Irimiciuc, 1978; Mangeron & Irimiciuc, 1980; Mangeron & Irimiciuc, 1981; Buzdugan *et al.*, 1979):

$$F_{a_1} = -C_1\dot{x}, \qquad F_{a_2} = -C_2\dot{x}. \tag{4}$$

Introducing of (3) and (4) in (2) leads to

$$m\ddot{x} + (C_1 + C_2)\dot{x} + (K_1 + K_2)x = mg + K_1 l_{10} + K_2 l_{20}. \tag{5}$$

3. Solution of the Dynamical Equation

The solution of eq. (5) is (Buzdugan *et al.*, 1979):

$$x = e^{-\lambda t}(x_1 \cos\omega\ t + x_2 \sin\omega\ t) + \frac{mg + K_1 l_{10} + K_2 l_{20}}{K_1 + K_2},$$

where λ and ω are given by (Buzdugan *et al.*, 1979; Teodorescu, 1984; Teodorescu, 1988; Teodorescu, 1997):

$$\lambda = \frac{C_1 + C_2}{2m}; \tag{6}$$

$$\omega = \sqrt{\frac{K_1 + K_2}{m} - \frac{(C_1 + C_2)^2}{4m^2}}, \tag{7}$$

while x_1 and x_2 are constants of integration.

In the hypothesis (1) the values of ω are real.

For the determination of equivalent damping constant one writes from relation (6):

$$\lambda = \frac{C_1 + C_2}{2m} = \frac{C_{ech}}{2m}. \tag{8}$$

In this way we obtain relation

$$C_{ech} = C_1 + C_2, \tag{9}$$

which is analogous with the equivalent elastic constant of parallel assembly.

The pulsation ω can be written in the form

$$\omega = \sqrt{\frac{K_{ech}}{m} - \frac{C_{ech}^2}{4m^2}},$$

where K_{ech} is the equivalent elastic constant (Buzdugan *et al.*, 1979):

$$K_{ech} = K_1 + K_2 . \tag{10}$$

4. Conclusions

In conclusion, the dynamical study can be made replacing the parallel assembly with an equivalent spring having the characteristics (elastic constant and damping constant) given by relations (9) and (10).

REFERENCES

Buzdugan Gh., Fetcu L., Radeş M., *Vibraţii mecanice.* Ed. Didactică şi Pedagogică, Bucureşti, 1979.

Mangeron D., Irimiciuc N., *Mecanica rigidelor cu aplicaţii în inginerie.* Vol. **I**, Ed. Tehnică, Bucureşti, 1978.

Mangeron D., Irimiciuc N., *Mecanica rigidelor cu aplicaţii în inginerie.* Vol. **II**, Ed. Tehnică, Bucureşti, 1980.

Mangeron D., Irimiciuc N., *Mecanica rigidelor cu aplicaţii în inginerie.* Vol. **III**, Ed. Tehnică, Bucureşti, 1981.

Teodorescu P.P., *Sisteme mecanice. Modele clasice*. Vol. **I**, Ed. Tehnică, Bucureşti, 1984.

Teodorescu P.P., *Sisteme mecanice. Modele clasice.* Vol. **II**, Ed. Tehnică, Bucureşti, 1988.

Teodorescu P.P., *Sisteme mecanice. Modele clasice.* Vol. **III**, Ed. Tehnică, Bucureşti, 1997.

DETERMINAREA CONSTANTEI DE AMORTIZARE ECHIVALENTE PENTRU UN ANSAMBLU DE ARCURI LEGATE ÎN PARALEL

(Rezumat)

Se consideră un sistem elastic compus din două arcuri legate în paralel cu o masă ataşată în poziţie verticală. Ecuaţia dinamică a mişcării în regim de vibraţii libere se compară cu ecuaţia dinamică a sistemului echivalent cu un singur arc şi se determină constanta de amortizare echivalentă a montajului paralel.